

# Quantum corrections in 2D SUSY $CP^{N-1}$ sigma model on noncommutative superspace

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**ABSTRACT:** We investigate quantum corrections in two-dimensional  $CP^{N-1}$  supersymmetric nonlinear sigma model on noncommutative superspace. We show that this model is renormalizable, the  $\mathcal{N}=2$  SUSY sector is not affected by the C-deformation and that the non(anti)commutativity parameter  $C^{\alpha\beta}$  receives infinite renormalization at one-loop order. And it is the renormalizability of the model at one-loop order.

**KEYWORDS:** Superspaces, Field Theories in Lower Dimensions, Sigma Models.

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## 1. Introduction

One recent development in quantum field theories is the emergence of new classes of field theories with some novel properties from the study of string theory. String theory with some of its field components having classical values on D-brane gives rise to new field theories in its low energy limits. In particular,  $\mathcal{N}=1$  noncommutative (NC) superspace considered some time ago [1] can be derived in this way from superstring theory with self-dual graviphoton background values [2–5].

In four dimensions, the non(anti)commutativity is introduced to  $\mathcal{N}=1$  superspace by deforming the anticommuting relations for superspace coordinates  $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$  as [4]

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \quad \{\theta^\alpha, \bar{\theta}^{\dot{\alpha}}\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0. \quad (1.1)$$

The noncommutativity (NC) parameter  $C^{\alpha\beta}$  is related to the (constant) graviphoton background value. Due to the non-zero anticommutator of  $\theta^\alpha$ , the field theory on the superspace keeps only a half of supersymmetry (SUSY),  $\mathcal{N}=1/2$  SUSY, some times also called deformed SUSY. Because  $\theta$  and  $\bar{\theta}$  are treated independently, the field theory on  $\mathcal{N}=1/2$  superspace can be defined only in Euclidian space and such field theories lack Hermiticity.

$\mathcal{N}=1/2$  SUSY field theories can be dealt with straightforwardly by modifying the product of superfields. The product is Weyl-ordered in  $\theta$  by using the Moyal product, which is defined by

$$f * g = f \exp\left(-\frac{1}{2} C^{\alpha\beta} \overleftarrow{Q}_\alpha \overrightarrow{Q}_\beta\right) g, \quad (1.2)$$

where  $Q_\alpha$  is the supercharge.

The Lagrangian of  $\mathcal{N}=1/2$  SUSY field theories consists of two parts, the original  $\mathcal{N}=1$  SUSY Lagrangian and the new terms containing  $C^{\alpha\beta}$  as a coupling constant. We refer to the two parts as the non-deformed SUSY sector and the C-deformation sector, respectively. The immediate question on  $\mathcal{N}=1/2$  SUSY field theories in perturbation is whether they are renormalizable, despite the fact that the new coupling constant  $C^{\alpha\beta}$  has a negative dimension (in mass). One may further ask whether good ultraviolet (UV) properties of field theories with (extended) SUSY are preserved after adding C-deformed terms. The first question has been answered positively for  $\mathcal{N}=1/2$  Wess-Zumino model [6–9] and  $\mathcal{N}=1/2$  supersymmetric gauge theory [9–13]. The NC parameter  $C^{\alpha\beta}$  has been shown to receive infinite renormalization. Quantum corrections to the non-deformed SUSY sector of these models are not affected by the C-deformation [10, 11].

$\mathcal{N}=2$   $CP^{N-1}$  supersymmetric nonlinear sigma models (SNLSM) [14, 15] are a low-energy effective theory of four-dimensional  $\mathcal{N}=1$  super Yang-Mills theories. Two-dimensional  $\mathcal{N}=2$  and  $\mathcal{N}=4$  SNLSM have remarkably good UV properties beyond perturbative renormalizability [16–19]. In non-perturbative aspects, two-dimensional SNLSM have instantons, like super Yang-Mills theories.

Since the Kähler potential of SNLSM is generally non-polynomial, the action of SNLSM on noncommutative superspace has infinitely many terms [20–23]. It is difficult to study the properties of such models even in perturbation. Fortunately the action takes a simple closed form in the case of  $CP^{N-1}$  SNLSM on NC superspace using the Kähler quotient [24].

The purpose of the present paper is to investigate one-loop quantum corrections in the two-dimensional  $CP^{N-1}$  SNLSM on NC superspace. In particular, we are interested in the loop effects on the non-deformed SUSY sector due to the new coupling  $C^{\alpha\beta}$  and vice versa. We have found that the C-deformation term of this model receives a divergent correction which is absorbed by the renormalization of the coupling constant  $\lambda$  and the NC parameter  $C^{\alpha\beta}$ .

## 2. 2D $CP^{N-1}$ SNLSM on NC superspace

$CP^{N-1}$  SNLSM in  $d = 2$  can be obtained from that in  $d = 4$  by dimensional reduction [25]. The same method can be used to obtain the  $CP^{N-1}$  SNLSM on NC superspace in  $d = 2$  [24].

We denote the scalar and fermion fields after solving the  $CP^{N-1}$  constraints by  $\varphi^a$ ,  $\bar{\varphi}^{\bar{a}}$  and  $\chi^a$ ,  $\bar{\chi}^{\bar{a}}$  ( $a = 1, 2, \dots, N-1$ ). The Lagrangian is written in terms of the component fields as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_C, \tag{2.1}$$

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{\lambda} g_{a\bar{b}} \partial_\mu \varphi^a \partial^\mu \bar{\varphi}^{\bar{b}} + \frac{1}{\lambda} i g_{a\bar{b}} \bar{\chi}_+^{\bar{b}} \gamma^\mu D_\mu \chi_+^a + \frac{1}{\lambda} i g_{a\bar{b}} \bar{\chi}_-^{\bar{b}} \gamma^\mu D_\mu \chi_-^a \\ & + \frac{1}{4\lambda} R_{a\bar{b}c\bar{d}} (\bar{\chi}_+^{\bar{b}} \gamma^\mu \chi_+^a) (\bar{\chi}_-^{\bar{d}} \gamma_\mu \chi_-^c), \end{aligned} \tag{2.2}$$

$$\mathcal{L}_C = \frac{1}{\lambda} g_{a\bar{b}} g_{c\bar{d}} (C^{11} \chi_+^a \chi_+^c - C^{22} \chi_-^a \chi_-^c) \epsilon^{\mu\nu} (\partial_\mu \bar{\varphi}^{\bar{b}}) (\partial_\nu \bar{\varphi}^{\bar{d}}). \tag{2.3}$$

Here  $\mathcal{L}_0$  is the non-deformed part, namely, the usual SUSY  $CP^{N-1}$  Lagrangian [14, 15],  $\mathcal{L}_C$  is the new term due to superspace noncommutativity.  $\chi_+^a, \chi_-^a$  are the two components of the 2D spinor  $\chi^a$ .  $g_{a\bar{b}}$  is the Fubini-Study metric on  $CP^{N-1}$ .  $\Gamma_{bc}^a$  and  $R_{a\bar{b}c\bar{d}}$  are the Christoffel symbol and the Riemann curvature tensor, respectively.  $D_\mu \chi_\pm^a$  is the covariant derivative. They are given by

$$g_{a\bar{b}} = \frac{(1 + \bar{\varphi}\varphi)\delta_{a\bar{b}} - \bar{\varphi}_a\varphi_{\bar{b}}}{(1 + \bar{\varphi}\varphi)^2}, \quad (2.4)$$

$$\Gamma_{bc}^a = g^{a\bar{d}}\partial_b g_{c\bar{d}}, \quad (2.5)$$

$$D_\mu \chi_\pm^a = \partial_\mu \chi_\pm^a + \Gamma_{bc}^a(\partial_\mu \varphi^b)\chi_\pm^c, \quad (2.6)$$

$$R_{a\bar{b}c\bar{d}} = -g_{a\bar{e}}\partial_c(g^{f\bar{e}}\partial_{\bar{d}}g_{f\bar{b}}) = g_{a\bar{b}}g_{c\bar{d}} + g_{a\bar{d}}g_{c\bar{b}}. \quad (2.7)$$

## 2.1 Background field method and Kähler normal coordinates

We evaluate the quantum corrections in our model in perturbation in the coupling constants  $\lambda$  and  $C^{\alpha\beta}$ . This can be made by means of the background field method. The familiar Riemann normal coordinates are not good for the present purpose, because they are not holomorphic on a Kähler manifold. Instead we should use the expansion in terms of Kähler normal coordinates [26], which are holomorphic coordinates. This expansion provides a manifestly covariant background field method preserving the complex structure on  $CP^{N-1}$ . We denote the background field by  $\varphi^a$ , the quantum field by  $\varphi_S^a$ , and the holomorphic coordinates on the  $CP^{N-1}$  by  $\xi^a$ .  $\varphi^a$ ,  $\partial_\mu \varphi^a$ ,  $g_{a\bar{b}}$ , and  $R_{a\bar{b}c\bar{d}}$  are written in terms of Kähler normal coordinates as

$$\varphi^a \rightarrow \varphi^a + \varphi_S^a(\sqrt{\lambda}\xi), \quad (2.8)$$

$$\partial_\mu \varphi_S^a(\sqrt{\lambda}\xi) = \sqrt{\lambda}D_\mu \xi^a - \lambda \frac{1}{2}\partial^\mu \bar{\varphi}^{\bar{b}} R_{c\bar{b}d}^a \xi^c \xi^d + O(\xi^3), \quad (2.9)$$

$$g_{a\bar{b}}(\varphi + \varphi_S(\sqrt{\lambda}\xi)) = g_{a\bar{b}} + \lambda R_{a\bar{b}c\bar{d}} \xi^c \bar{\xi}^{\bar{d}} + O(\xi^3), \quad (2.10)$$

$$R_{a\bar{b}c\bar{d}}(\varphi + \varphi_S(\sqrt{\lambda}\xi)) = R_{a\bar{b}c\bar{d}} + \lambda g^{e\bar{f}}(R_{e(\bar{b}g\bar{d}}R_{a\bar{h}c\bar{f}})_{\bar{f}} - R_{e\bar{b}g\bar{d}}R_{a\bar{h}c\bar{f}})\xi^g \bar{\xi}^{\bar{h}} + O(\xi^3). \quad (2.11)$$

In the following computation of one-loop effects, we need the terms of second order in the fluctuations  $\xi^a$  of the Lagrangian (2.2), (2.3). They are given by

$$\begin{aligned} \mathcal{L}_0^{(2)} &= g_{a\bar{b}}D_\mu \xi^a D^\mu \bar{\xi}^{\bar{b}} \\ &+ R_{a\bar{b}c\bar{d}} \left( \partial_\mu \varphi^a \partial^\mu \bar{\varphi}^{\bar{b}} \xi^c \bar{\xi}^{\bar{d}} - \frac{1}{2}\partial_\mu \bar{\varphi}^{\bar{b}} \partial^\mu \varphi^{\bar{d}} \xi^a \xi^c - \frac{1}{2}\partial_\mu \varphi^a \partial^\mu \bar{\varphi}^{\bar{c}} \bar{\xi}^{\bar{b}} \bar{\xi}^{\bar{d}} \right) \\ &+ iR_{a\bar{b}c\bar{d}}(\bar{\chi}_+^{\bar{b}}\gamma^\mu D_\mu \chi_+^a \xi^c \bar{\xi}^{\bar{d}} + \bar{\chi}_+^{\bar{b}}\gamma^\mu \chi_+^c D_\mu \xi^a \bar{\xi}^{\bar{d}}) \\ &+ iR_{a\bar{b}c\bar{d}}(\bar{\chi}_-^{\bar{b}}\gamma^\mu D_\mu \chi_-^a \xi^c \bar{\xi}^{\bar{d}} + \bar{\chi}_-^{\bar{b}}\gamma^\mu \chi_-^c D_\mu \xi^a \bar{\xi}^{\bar{d}}) \\ &+ g^{e\bar{f}}(R_{e(\bar{b}g\bar{d}}R_{a\bar{h}c\bar{f}})_{\bar{f}} - R_{e\bar{b}g\bar{d}}R_{a\bar{h}c\bar{f}})\xi^g \bar{\xi}^{\bar{h}}(\bar{\chi}_+^{\bar{b}}\gamma^\mu \chi_+^a)(\bar{\chi}_-^{\bar{d}}\gamma^\mu \chi_-^c), \quad (2.12) \end{aligned}$$

$$\mathcal{L}_C^{(2)} = \left\{ g_{a\bar{b}}g_{c\bar{d}} \left( D_\mu \bar{\xi}^{\bar{b}} D_\nu \bar{\xi}^{\bar{d}} - \frac{1}{2}\partial_\mu \varphi^e \partial_\nu \bar{\varphi}^{\bar{d}} R_{f\bar{e}g}^{\bar{b}} \bar{\xi}^{\bar{f}} \bar{\xi}^{\bar{g}} \right. \right.$$

$$\begin{aligned}
 & -\frac{1}{2}\partial_\nu\varphi^e\partial_\mu\bar{\varphi}^{\bar{b}}R_{\bar{f}e\bar{g}}^{\bar{d}}\bar{\xi}^{\bar{f}}\bar{\xi}^{\bar{g}}) + g_{ab}\partial_\mu\bar{\varphi}^{\bar{b}}\partial_\nu\bar{\varphi}^{\bar{d}}R_{c\bar{d}e\bar{f}}\xi^e\bar{\xi}^{\bar{f}} \\
 & + g_{c\bar{d}}\partial_\mu\bar{\varphi}^{\bar{b}}\partial_\nu\bar{\varphi}^{\bar{d}}R_{a\bar{b}e\bar{f}}\xi^e\bar{\xi}^{\bar{f}} \left. \vphantom{R_{a\bar{b}e\bar{f}}} \right\} \epsilon^{\mu\nu} (C^{11}\chi_+\chi_+^c - C^{22}\chi_-\chi_-^c). \tag{2.13}
 \end{aligned}$$

### 3. One-loop perturbative corrections

We begin by noting that all terms in the C-deformation part  $\mathcal{L}_C$  contain  $\chi^a\chi^b$  and are non-Hermitian. In perturbation, only the combination  $\chi^a\bar{\chi}^b$  is contracted in the Wick's theorem; the combination  $\chi^a\chi^b$  is not contracted. Hence the C-deformed terms with  $\chi^a\chi^b$  cannot appear as a part of internal lines. They can only appear as external lines. This means that there appear no loop corrections due to the new term  $\mathcal{L}_C$  to the non-deformed part of the effective action. The same property holds for super Yang-Mills theories [10, 11].

#### 3.1 Non-deformed sector

Because of the absence of loop corrections due to  $\mathcal{L}_C$ , the loop corrections to the non-deformed SUSY sector in our model are the same as those in the ordinary model. In particular, the  $\beta$ -function of  $\lambda$  is not affected by C-deformation. For later use we summarize the previous result on the renormalization of the non-deformed SUSY sector [17, 18]. We use the dimensional regularization, setting  $\epsilon = 1 - d/2$ , where  $d$  is the space-time dimension. The divergent one-loop correction to the non-deformed sector is written as that to  $\mathcal{L}_0$ .

$$\begin{aligned}
 \delta\mathcal{L}_{\text{Boson}} &= R_{a\bar{b}c\bar{d}}\partial_\mu\varphi^a\partial^\mu\bar{\varphi}^{\bar{b}}\dot{\xi}^c\dot{\xi}^{\bar{d}} \\
 &= \frac{1}{2\epsilon}\frac{1}{2\pi}R_{a\bar{b}}\partial_\mu\varphi^a\partial^\mu\bar{\varphi}^{\bar{b}} \\
 &= \frac{1}{2\epsilon}\frac{\lambda N}{2\pi}\mathcal{L}_{\text{Boson}}. \tag{3.1}
 \end{aligned}$$

Here  $\cdot$  (or  $\dot{\cdot}$ ) over a pair of fields  $\xi^a(x)$  mean that they are contracted. We have used the relation which holds for  $CP^{N-1}$ ,

$$R_{a\bar{b}} = Ng_{a\bar{b}}. \tag{3.2}$$

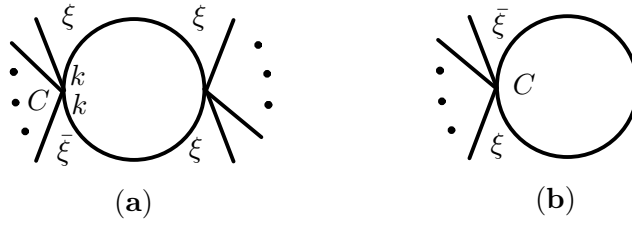
The bare quantities will be denoted by the suffix 0.  $g_{a\bar{b}}/\lambda_0$  can be expressed in terms of one-loop order renormalized quantities as

$$\begin{aligned}
 \frac{g_{a\bar{b}}}{\lambda_0} &= \mu^{2\epsilon} \left\{ \frac{g_{a\bar{b}}}{\lambda} + \frac{1}{2\epsilon} \left( -\frac{N}{2\pi} g_{a\bar{b}} \right) + \dots \right\} \\
 &= \mu^{2\epsilon} \frac{g_{a\bar{b}}}{\lambda} \left( 1 - \frac{1}{2\epsilon} \frac{\lambda N}{2\pi} \right), \tag{3.3}
 \end{aligned}$$

where  $\mu$  is the scale parameter.

#### 3.2 C-deformed sector

We now evaluate the loop corections to the other coupling constant  $C^{\alpha\beta}$ . There are two graphs which give possibly divergent contributions to the C-deformed sector. They are



**Figure 1:** Divergent one-loop corections to the C-deformed sector. The solid internal lines stand for the  $\xi$ -propagator, the dots denote background-dependent vertices, such as (3.4).

shown in Fig.1. Other one-loop graphs contributing to the C-deformed sector are finite. In Fig.1 the external lines represent a set of background fields, such as

$$\delta_{a\bar{b}}\delta_{c\bar{d}}\epsilon^{\mu\nu}\partial_\mu\bar{\varphi}^{\bar{b}}\partial_\nu\bar{\varphi}^{\bar{d}}C^{11}\chi_+\chi_+^c. \quad (3.4)$$

The loop graph 1.a involves two vertices, one containing  $D_\mu\bar{\xi}^{\bar{b}}D_\nu\bar{\xi}^{\bar{d}}$  from  $\mathcal{L}_C^{(2)}$  and another containing  $\xi^a\xi^c$  from  $\mathcal{L}_{\text{Boson}}^{(2)}$ .

$$\mathcal{L}_{\text{Boson}}^{(2)} = \dots - \frac{1}{2}R_{a\bar{b}c\bar{d}}\partial_\mu\bar{\varphi}^{\bar{b}}\partial^\mu\bar{\varphi}^{\bar{d}}\xi^a\xi^c + \dots, \quad (3.5)$$

$$\mathcal{L}_C^{(2)} = \dots + g_{a\bar{b}}g_{c\bar{d}}\epsilon^{\mu\nu}(C^{11}\chi_+\chi_+^c - C^{22}\chi_-\chi_-^c)D_\mu\bar{\xi}^{\bar{b}}D_\nu\bar{\xi}^{\bar{d}} + \dots. \quad (3.6)$$

The graph 1.a contributes to the effective action the following term.

$$\begin{aligned} \delta\mathcal{L}_C^{(a)} &= (\dots) \times g_{a\bar{b}}g_{c\bar{d}}R_{i\bar{j}k\bar{l}}\partial^\sigma\bar{\varphi}^{\bar{j}}\partial^\sigma\bar{\varphi}^{\bar{l}}\epsilon^{\mu\nu}D_\mu\bar{\xi}^{\bar{b}}D_\nu\bar{\xi}^{\bar{d}} \times \xi^i\xi^k \\ &= (\dots) \times \epsilon_{\mu\nu}(I^{\mu\nu} + \text{finite}), \end{aligned} \quad (3.7)$$

where  $I^{\mu\nu}$  is given by

$$\begin{aligned} I^{\mu\nu} &= \int_0^1 dx \int d^2k \frac{k^\mu k^\nu}{\{k^2 + M^2(x)\}^2} \\ &\sim \frac{1}{\epsilon}\delta^{\mu\nu}. \end{aligned} \quad (3.8)$$

Here  $M^2(x)$  does not contain  $k^\mu$ . We note that  $I^{\mu\nu}$  is symmetric in  $\mu, \nu$  and hence

$$\delta\mathcal{L}_C^{(a)} \sim \epsilon_{\mu\nu}I^{\mu\nu} = 0. \quad (3.9)$$

The loop graph 1.b involves a single vertex, the one containing  $\xi^i\bar{\xi}^{\bar{j}}$  from  $\mathcal{L}_C^{(2)}$ .

$$\mathcal{L}_C^{(2)} = (g_{a\bar{b}}R_{c\bar{d}i\bar{j}} + g_{c\bar{d}}R_{a\bar{b}i\bar{j}})\partial_\mu\bar{\varphi}^{\bar{b}}\partial_\nu\bar{\varphi}^{\bar{d}}\epsilon^{\mu\nu}(C^{11}\chi_+\chi_+^c - C^{22}\chi_-\chi_-^c)\xi^i\bar{\xi}^{\bar{j}} + \dots. \quad (3.10)$$

It's contribution to the effective action is computed to be

$$\delta\mathcal{L}_C^{(b)} = (g_{a\bar{b}}R_{c\bar{d}i\bar{j}} + g_{c\bar{d}}R_{a\bar{b}i\bar{j}})\partial_\mu\bar{\varphi}^{\bar{b}}\partial_\nu\bar{\varphi}^{\bar{d}}\epsilon^{\mu\nu}(C^{11}\chi_+\chi_+^c - C^{22}\chi_-\chi_-^c)\xi^i\bar{\xi}^{\bar{j}}$$

$$= \frac{1}{2\epsilon} \frac{N}{2\pi} (g_{ab}g_{c\bar{d}} + g_{c\bar{d}}g_{ab}) \partial_\mu \bar{\varphi}^b \partial_\nu \bar{\varphi}^{\bar{d}} \epsilon^{\mu\nu} (C^{11} \chi_+^a \chi_+^c - C^{22} \chi_-^a \chi_-^c). \quad (3.11)$$

We thus have

$$\delta \mathcal{L}_C = \frac{1}{\epsilon} \frac{\lambda N}{2\pi} \mathcal{L}_C. \quad (3.12)$$

To summarize, at one-loop, only the loop graph of 1.b gives a divergent contribution to the C-deformed sector. The result is proportional to  $\mathcal{L}_C$ , and hence it can be eliminated by the counter term. It assures the renormalizability of the model at one-loop order.

### 3.3 UV divergences in the C-deformed sector and renormalization

We decompose  $C^{\alpha\beta}$  into renormalization part and constant part by writing

$$C^{\alpha\beta} = \gamma \tilde{C}^{\alpha\beta}. \quad (3.13)$$

The dimensionless coupling constant  $\gamma$  receives renormalization and  $\tilde{C}^{\alpha\beta}$  is set to constant, in the Lagrangian (2.3). We introduce  $T^{a\bar{b}c\bar{d}}$  as

$$T^{a\bar{b}c\bar{d}} \equiv \partial_\mu \bar{\varphi}^b \partial_\nu \bar{\varphi}^{\bar{d}} \epsilon^{\mu\nu} \{ \tilde{C}^{11} \chi_+^a \chi_+^c - \tilde{C}^{22} \chi_-^a \chi_-^c \}. \quad (3.14)$$

The counter term of  $\mathcal{L}_C$  is  $(\mathcal{L}_C)_0 - \mathcal{L}_C$ . By using the renormalization result (3.3), we have

$$\begin{aligned} \mathcal{L}_{C,\text{ct}} = (\mathcal{L}_C)_0 - \mathcal{L}_C &= \lambda_0 \gamma_0 \frac{g_{ab}}{\lambda} \frac{g_{c\bar{d}}}{\lambda} \left( 1 - \frac{1}{2\epsilon} \frac{\lambda N}{2\pi} \right)^2 T^{a\bar{b}c\bar{d}} - \lambda \gamma \frac{g_{ab}}{\lambda} \frac{g_{c\bar{d}}}{\lambda} T^{a\bar{b}c\bar{d}} \\ &= \left\{ \lambda_0 \gamma_0 \left( 1 - \frac{1}{2\epsilon} 2 \frac{\lambda N}{2\pi} \right) - \lambda \gamma \right\} \frac{g_{ab}}{\lambda} \frac{g_{c\bar{d}}}{\lambda} T^{a\bar{b}c\bar{d}} \\ &= \left\{ \frac{\lambda_0 \gamma_0}{\lambda \gamma} \left( 1 - \frac{1}{2\epsilon} 2 \frac{\lambda N}{2\pi} \right) - 1 \right\} \mathcal{L}_C. \end{aligned} \quad (3.15)$$

It is added to cancel the infinity from the loop graph, which we have computed above.

$$\begin{aligned} \mathcal{L}_{C,\text{ct}} + \delta \mathcal{L}_C &= \left( \frac{\lambda_0 \gamma_0}{\lambda \gamma} - 1 \right) \left( 1 - \frac{1}{2\epsilon} 2 \frac{\lambda N}{2\pi} \right) \mathcal{L}_C \\ &= 0. \end{aligned} \quad (3.16)$$

Thus the renormalization of  $\gamma$  is fixed by the condition

$$\lambda_0 \gamma_0 = \lambda \gamma. \quad (3.17)$$

We have found that the NC parameter  $C^{\alpha\beta}$  (or  $\gamma$ ) is renormalized at one-loop order as

$$\gamma_0 / \gamma = (\lambda_0 / \lambda)^{-1} = \mu^{2\epsilon} \left( 1 - \frac{1}{2\epsilon} \frac{\lambda N}{2\pi} \right). \quad (3.18)$$

### 3.4 $\beta$ -functions

The ordinary 2D SUSY  $CP^{N-1}$  model has good UV properties. We now study the UV properties of the deformed 2D SUSY  $CP^{N-1}$  model, in particular its  $\beta$ -functions. The  $\beta$ -function of the coupling constant  $\lambda$  is obtained from eq.(3.3).

$$\beta_\lambda = \mu \frac{\partial}{\partial \mu} \lambda = -\frac{N}{2\pi} \lambda^2. \quad (3.19)$$

For the purpose of perturbative computation we have defined the fields by setting

$$\varphi^a, \chi^a \rightarrow \sqrt{\lambda} \varphi^a, \quad \sqrt{\lambda} \chi^a, \quad (3.20)$$

$$g_{a\bar{b}} = \frac{(1 + \lambda \bar{\varphi} \varphi) \delta_{a\bar{b}} - \lambda \bar{\varphi}_a \varphi_{\bar{b}}}{(1 + \lambda \bar{\varphi} \varphi)^2}. \quad (3.21)$$

$\mathcal{L}_C$  is then written as

$$\mathcal{L}_C = \lambda \gamma g_{a\bar{b}} g_{c\bar{d}} (\tilde{C}^{11} \chi_+^a \chi_+^c - \tilde{C}^{22} \chi_-^a \chi_-^c) \epsilon^{\mu\nu} (\partial_\mu \bar{\varphi}^{\bar{b}}) (\partial_\nu \bar{\varphi}^{\bar{d}}). \quad (3.22)$$

We take  $\lambda\gamma$  instead of  $\gamma$  as the new coupling constant. The  $\beta$ -function of  $\lambda\gamma$  vanishes.

$$\beta_{\lambda\gamma} = \mu \frac{\partial}{\partial \mu} (\lambda\gamma) = 0. \quad (3.23)$$

We note that  $\lambda = 0$  is the UV fixed line of the theory in the  $\lambda - \gamma\lambda$  plane.

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### References

- [1] S. Ferrara and M.A. Lledo, *Some aspects of deformations of supersymmetric field theories*, *JHEP* **05** (2000) 008 [[hep-th/0002084](#)];  
D. Klemm, S. Penati and L. Tamassia, *Non(anti)commutative superspace*, *Class. and Quant. Grav.* **20** (2003) 2905 [[hep-th/0104190](#)];  
S. Ferrara, M.A. Lledo and O. Macia, *Supersymmetry in noncommutative superspaces*, *JHEP* **09** (2003) 068 [[hep-th/0307039](#)].
- [2] H. Ooguri and C. Vafa, *The C-deformation of gluino and non-planar diagrams*, *Adv. Theor. Math. Phys.* **7** (2003) 53 [[hep-th/0302109](#)]; *Gravity induced C-deformation*, *Adv. Theor. Math. Phys.* **7** (2004) 405 [[hep-th/0303063](#)].
- [3] J. de Boer, P.A. Grassi and P. van Nieuwenhuizen, *Non-commutative superspace from string theory*, *Phys. Lett.* **B 574** (2003) 98 [[hep-th/0302078](#)].
- [4] N. Seiberg, *Noncommutative superspace,  $N = 1/2$  supersymmetry, field theory and string theory*, *JHEP* **06** (2003) 010 [[hep-th/0305248](#)].



- [5] N. Berkovits and N. Seiberg, *Superstrings in graviphoton background and  $N = 1/2 + 3/2$  supersymmetry*, *JHEP* **07** (2003) 010 [[hep-th/0306226](#)].
- [6] M.T. Grisaru, S. Penati and A. Romagnoni, *Two-loop renormalization for nonanticommutative  $N = 1/2$  supersymmetric WZ model*, *JHEP* **08** (2003) 003 [[hep-th/0307099](#)].
- [7] R. Britto and B. Feng,  *$N = 1/2$  Wess-Zumino model is renormalizable*, *Phys. Rev. Lett.* **91** (2003) 201601 [[hep-th/0307165](#)].
- [8] A. Romagnoni, *Renormalizability of  $N = 1/2$  Wess-Zumino model in superspace*, *JHEP* **10** (2003) 016 [[hep-th/0307209](#)].
- [9] D. Berenstein and S.J. Rey, *Wilsonian proof for renormalizability of  $N = 1/2$  supersymmetric field theories*, *Phys. Rev. D* **68** (2003) 121701 [[hep-th/0308049](#)].
- [10] M. Alishahiha, A. Ghodsi and N. Sadooghi, *One-loop perturbative corrections to non(anti)commutativity parameter of  $N = 1/2$  supersymmetric  $U(N)$  gauge theory*, *Nucl. Phys. B* **691** (2004) 111 [[hep-th/0309037](#)].
- [11] I. Jack, D.R.T. Jones and L.A. Worthy, *One-loop renormalisation of  $N = 1/2$  supersymmetric gauge theory*, *Phys. Lett. B* **611** (2005) 199 [[hep-th/0412009](#)].
- [12] T. Araki, K. Ito and A. Ohtsuka, *Supersymmetric gauge theories on noncommutative superspace*, *Phys. Lett. B* **573** (2003) 209 [[hep-th/0307076](#)].
- [13] S. Terashima and J.T. Yee, *Comments on noncommutative superspace*, *JHEP* **12** (2003) 053 [[hep-th/0306237](#)].
- [14] A. D’Adda, P. Di Vecchia and M. Lüscher, *Confinement and chiral symmetry breaking in  $CP^{n-1}$  models with quarks*, *Nucl. Phys. B* **152** (1979) 125.
- [15] E. Cremmer and J. Scherk, *The supersymmetric nonlinear sigma model in four-dimensions and its coupling to supergravity*, *Phys. Lett. B* **74** (1978) 341.
- [16] M.T. Grisaru, A.E.M. van de Ven and D. Zanon, *Four loop beta function for the  $N = 1$  and  $N = 2$  supersymmetric nonlinear sigma model in two-dimensions*, *Phys. Lett. B* **173** (1986) 423.
- [17] L. Alvarez-Gaumé, D.Z. Freedman and S. Mukhi, *The background field method and the ultraviolet structure of the supersymmetric nonlinear sigma model*, *Ann. Phys. (NY)* **134** (1981) 85.
- [18] L. Alvarez-Gaumé and D.Z. Freedman, *Geometrical structure and ultraviolet finiteness in the supersymmetric sigma model*, *Commun. Math. Phys.* **80** (1981) 443.
- [19] A.Y. Morozov, A.M. Perelomov and M.A. Shifman, *Exact Gell-Mann-Low function of supersymmetric Kähler sigma models*, *Nucl. Phys. B* **248** (1984) 279.
- [20] B. Chandrasekhar and A. Kumar,  *$D = 2, N = 2$ , supersymmetric theories on non(anti)commutative superspace*, *JHEP* **03** (2004) 013 [[hep-th/0310137](#)];  *$D = 2, N = 2$  supersymmetric sigma models on non(anti)commutative superspace*, *Phys. Rev. D* **70** (2004) 125003 [[hep-th/0408184](#)].
- [21] O.D. Azorkina, A.T. Banin, I.L. Buchbinder and N.G. Pletnev, *Generic chiral superfield model on nonanticommutative  $N = 1/2$  superspace*, *Mod. Phys. Lett. A* **20** (2005) 1423 [[hep-th/0502008](#)].

- [22] L. Alvarez-Gaumé and M.A. Vazquez-Mozo, *On nonanticommutative  $N = 2$  sigma-models in two dimensions*, *JHEP* **04** (2005) 007 [[hep-th/0503016](#)].
- [23] B. Chandrasekhar,  *$D = 2$ ,  $N = 2$  supersymmetric sigma models on non(anti)commutative superspace*, *Phys. Rev.* **D 70** (2004) 125003 [[hep-th/0408184](#)];  *$N = 2$  sigma-model action on non(anti)commutative superspace*, *Phys. Lett.* **B 614** (2005) 207 [[hep-th/0503116](#)].
- [24] T. Inami and H. Nakajima, *Supersymmetric  $CP^n$  sigma model on noncommutative superspace*, *Prog. Theor. Phys.* **111** (2004) 961 [[hep-th/0402137](#)].
- [25] S. Aoyama and M. Tonin, *The dimensional regularization of chiral gauge theories and generalized Slavnov-Taylor identities*, *Nucl. Phys.* **B 179** (1981) 293.
- [26] K. Higashijima and M. Nitta, *Kähler normal coordinate expansion in supersymmetric theories*, *Prog. Theor. Phys.* **105** (2001) 243 [[hep-th/0006027](#)];  
K. Higashijima, E. Itou and M. Nitta, *Normal coordinates in Kähler manifolds and the background field method*, *Prog. Theor. Phys.* **108** (2002) 185 [[hep-th/0203081](#)].